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Resonant modes and inter-well coupling in photonic quantum well with negative index materials

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Abstract. The transfer matrix method was used to study the resonant modes in photonic quantum well by stacking different photonic crystals consisting of positive index materials and negative index materials. The eigenfrequency equation for the resonant modes is derived. It is found that these resonant modes are omnidirectional, and the number of resonant modes is equal to the period number of photonic quantum wells. Moreover, the resonant modes become N-fold splitting in the N photonic quantum wells. The splitting intervals increase with the deceasing of photonic barrier thickness due to the coupling among the wells.

PACS. 42.70.Qs Photonic bandgap materials – 41.20.Jb Electromagnetic wave propagation; radiowave propagation – 42.79.Ci Filters, zone plates, and polarizers

1 Introduction

Since the pioneering work of Yablonovitch and John [1,2], photonic crystals (PCs) have attracted considerable attention due to potential applications of photonic band gap. It is well known that electronic quantum well (QW) is formed by electronic band mismatch. If the energy band of the well layer is submerged into the band gap of the barrier layer, the electronic energies in electronic quantum well are quantised due to quantum confinement effects. Using an electronic quantum well as an analogy, photonic quantum well has been proposed by inserting a photonic well into photonic barriers [3–8]. Due to the confinement effect of photonic barriers, quantised confined modes in such structures are predicted in these studies. If the frequency is tuned to the frequency of the quantised confined modes, the light will be transmitted in manner of resonance, leading to the multiple channeled filtering phenomena [9,10]. So, the quantised confined modes are also called the resonant modes. It must be noted here that resonant modes in conventional photonic QW come into being owing to the confinement effect of Bragg gap. Therefore the resonant modes in the Bragg gap depend strongly on incident angles and polarisations because of multiple scattering. For this reason, the phenomenon has only been used for filters under the limit of normal incidence. However, if we use a novel gap originated from the mechanism beyond Bragg scatting as the photonic barrier, we may overcome the shortcomings mentioned above.

It is shown that one-dimensional photonic crystals (1DPCs) fabricated by positive index materials (PIMs) and negative index materials (NIMs) have a new type of band gap called zero-averaged refractive index (zero-$\bar{n}$) gap [11]. The properties of the zero-$\bar{n}$ gap are independent of incident angles and polarisations [12], which differs from that of Bragg gap. Omnidirectional defect modes are found in the zero-$\bar{n}$ gap when defects are introduced [13–16]. It is expected that the resonant modes of photonic QW where the zero-$\bar{n}$ gap acts as photonic barrier can show the omnidirectional properties. In this paper we investigate the properties of resonant modes in the zero-$\bar{n}$ gap and inter-well coupling among multiple photonic QW containing NIMs by means of the transfer matrix methods. The paper is organised as follows. In Section 2 we derive the eigenfrequncy equation for the resonant modes of photonic QW containing NIMs and study the dependence of resonant modes on the incident angles and polarization. In Section 3 we discussed the mutual coupling among N photonic QW. It is shown that the resonant modes become N-fold splitting and the splitting interval increases with the decrease of photonic barrier thickness. Finally, we conclude the report in Section 4.

2 Eigenfrequency equation for the resonant modes in photonic QW

The photonic QW in our research is stacked by two different 1DPCs $(AB)_m$ and $(CD)_n$ which are shown in Figure 1. $A(C)$ and $B(D)$ indicate the PIMs and NIMs respectively. $m(n)$ is the period number. The permittivity

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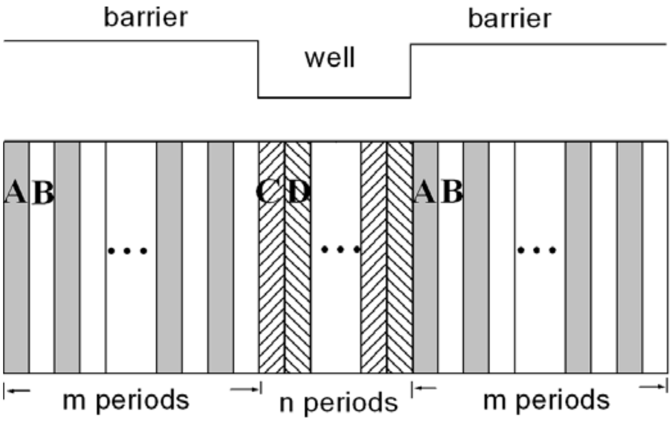

Fig. 1. Schematic of photonic quantum well structure by stacking two different 1D photonic crystals $(AB)_m$ and $(CD)_n$.

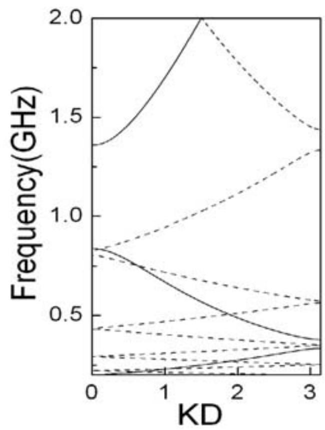

Fig. 2. The dispersion relation of frequency versus the Bloch wave number for $(AB)_m$ (the solid line) and for $(CD)_n$ (the dashed line) with the following parameters: $\varepsilon_{A(C)} = 9.0$, $\mu_{A(C)} = 1.0$, $\varepsilon_0 = \mu_0 = 1.0$, $\alpha = 900$ and $\beta = 400$; $d_A = 16$ mm, $d_B = 4$ mm, $d_C = 2.5$ mm and $d_D = 17.5$ mm.

and permeability of $A(C)$ is $\varepsilon_{A(C)} = 9.0$ and $\mu_{A(C)} = 1.0$. $B(D)$ are assumed to be isotropic and dispersive, with effective ε and μ given by

$$\varepsilon_{B(D)} = \varepsilon_0 - \frac{\alpha}{\omega^2} \tag{1}$$

$$\mu_{B(D)} = \mu_0 - \frac{\beta}{\omega^2}. \tag{2}$$

The dispersion relations can be realised in a composite made of periodically LC loaded transmission lines. In equations (1) and (2), ω is the angular frequency measured in gigahertz, ε_0 and μ_0 represent permittivity and permeability of an unperturbed transmission line, α and β are circuit parameters and can be modulated with great freedom. In our calculation, we choose the parameters $\varepsilon_0 = \mu_0 = 1.0$, $\alpha = 900$ and $\beta = 400$ as that in reference [17]. The thicknesses of A, B, C and D is $d_A = 16$ mm, $d_B = 4$ mm, $d_C = 2.5$ mm and $d_D = 17.5$ mm respectively. Photonic band structure for $(AB)_m$ and $(CD)_n$ is shown in Figure 2 on normal incidence. It is obvious that there is a frequency interval from 0.83 to 1.35 GHz, which is the zero-$\bar{n}$ gap [10,11] for $(AB)_m$ but in the allowed band for $(CD)_n$. For the frequency in this interval, the structure is analogous with an electronic QW of the semiconductor. The $(AB)_m$ acts as a barrier while the $(CD)_n$ acts as a well. The electromagnetic wave is evanescent in the $(AB)_m$ region while it takes a form of travelling wave in the $(CD)_n$ region.

In order to derive the eigenfrequency equation for the resonant modes, we presume that $m \to \infty$ so the photonic QW can be regarded as two semi-infinite PCs doped by an array of defects. Let a monochromatic plane, either a transverse electric (TE) or a transverse magnetic (TM) wave be incident along the z direction from vacuum onto the considered structure. The electric and magnetic fields in z and $z + \Delta z$ in the same layer can be related via a transfer matrix [18]

$$M_i(\Delta z) = \begin{pmatrix} \cos(k_i \Delta z) & -\frac{1}{\sigma_i}\sin(k_i \Delta z) \\ \sigma_i \sin(k_i \Delta z) & \cos(k_i \Delta z) \end{pmatrix} (i = A, B, C, D), \tag{3}$$

where $k_i = \frac{\omega}{c}\delta\left(\varepsilon_i\mu_i - \sin^2\theta\right)^{1/2}$, $\delta = \pm 1$, we let $\delta = +1$ for PIMs and $\delta = -1$ for NIMs (when ε_i and μ_i are all negative, k_i is negative too). c is the light speed in vacuum and θ is the incident angle; $\sigma_i = \frac{k_i c}{\omega \mu_i}$ for TE wave and $\sigma_i = \frac{k_i c}{\omega \varepsilon_i}$ for TM wave. For perfect 1DPCs, according to the Bloch theorem, the dispersive relation can be written as

$$\cos(Kd) = \frac{1}{2}\mathrm{Tr}\left[M_B(d_B)M_A(d_A)\right] = \frac{1}{2}\mathrm{Tr}\,Q, \tag{4}$$

where K is the Bloch wave number and d is the period of PCs AB. According to equation (4) the allowed band and the forbidden band occurs when $|1/2\mathrm{Tr}\,Q| < 1$ or $|1/2\mathrm{Tr}\,Q| > 1$, respectively.

When an array of $(CD)_n$ is sandwiched between the two semi-infinite PCs, the resonant mode will be found in the forbidden gap. For the resonant modes, the electromagnetic waves inside the two semi-infinite PCs are evanescent. These evanescent waves can be connected with each other via a transfer matrix

$$W = \left[M_D(d_d)M_C(d_C)\right]^n. \tag{5}$$

After some algebra calculation, we obtain the eigenfrequency equation for the resonant modes

$$xy_2W_{11} + y_1y_2W_{12} + x^2W_{21} + xy_1W_{22} = 0, \tag{6}$$

where W_{ij} are the elements of matrix W and $x = Q_{12}$, $y_1 = Q_{22} - \Lambda$, $y_2 = Q_{11} - \Lambda$, $\Lambda = \eta(1 - \sqrt{1 - 11\eta^2}$ and $\eta = 1/2\mathrm{Tr}\,Q$, Q_{ij} are the elements of matrix Q.

Firstly, we let $\theta = 0°$. Through solving the eigenfrequency equation by means of Bisection Method we can find the relationships between resonant modes and the period numbers of $(CD)_n$ in the well region, which is plotted in Figure 3. We can see that the number of resonant modes are equal to the periods of well. This is similar to that of the conventional photonic QW [9].

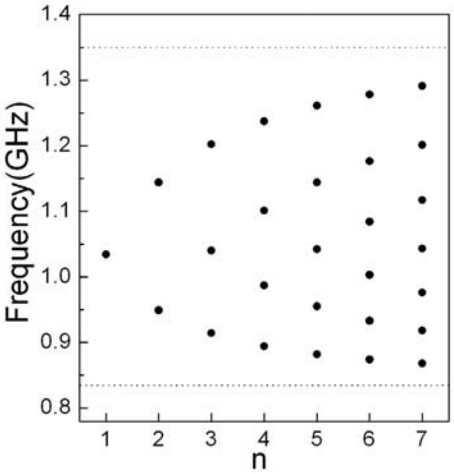

Fig. 3. Variation of the resonant modes in the forbidden band as a function of the period numbers of $(CD)_n$ in the well region. The dotted line represents the band edge. The parameters are the same as those in Figure 2.

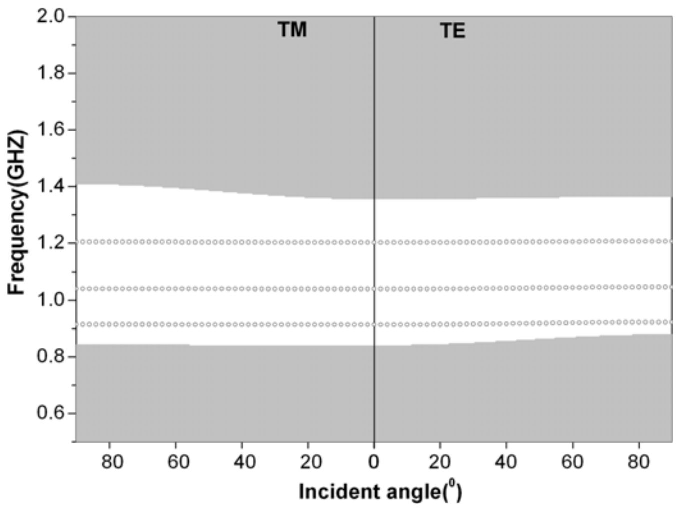

Fig. 4. Dependence of the photonic band gap and the resonant modes on the incident angle in infinite structure $(AB)_m\,(CD)_n\,(AB)_m$, with $n = 3$. Other parameters are the same as those in Figure 2.

We then investigated the dependence of resonant modes inside the zero-$\bar{n}$ gap on the incident angle. The dispersion relation of the photonic band gap and these resonant modes are shown in Figure 4, in which, we select the periods of well $n = 3$. It can be seen from the figures that three resonant modes appear in zero-$\bar{n}$ gap at frequencies about 0.913, 1.041 and 1.203 GHz, respectively. As shown in Figure 4, the three resonant modes inside the zero-$\bar{n}$ gap are almost independent of the incident angles and polarisations. The weak incident angle dependence of the resonant mode inside zero-$\bar{n}$ gap may be useful in applications, such as omnidirectional filters with multi-channels, whose number of channel and frequency interval can be varied conveniently by adjusting the period numbers of $(CD)_n$ in the well region.

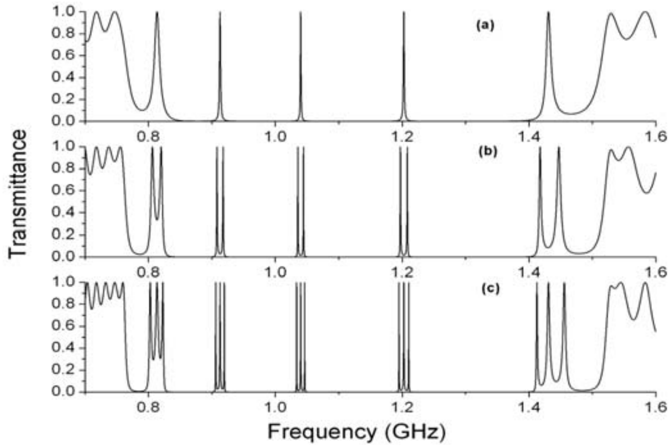

Fig. 5. The transmittance through the $[(AB)_4(CD)_3]^N(AB)_4$ structure: (a) for $N = 1$, (b) for $N = 2$ and (c) for $N = 3$ respectively. Other parameters are the same as those in Figure 2.

3 Inter-well coupling in multiple photonic QW structure

In this section, we study the inter-well coupling in multiple photonic QW structure. We denote the structure as $[(AB)_m(CD)_n]^N(AB)_m$, where N is the number of photonic wells. Figure 5 shows the transmittance through $[(AB)_4(CD)_3]^N(AB)_4$ structure under different numbers of the photonic well. You can see from Figure 5a that there are three resonant tunneling modes in the gap, which agree with the above results obtained by solving eigenfrequency equation. With the increasing number of photonic wells, each mode splits two in Figure 5b and three in Figure 5c. We can come to the conclusion that the modes become N-fold splitting if there are N photonic wells. The splitting properties are analogous to that of electrons in a semiconductor superlattice [19]. We can explain the phenomena by the following inter-well coupling mechanism. If the N photonic wells are separated from each other with a large enough barrier width, there is no interaction between adjacent wells and the multiple photonic QW will have N-fold degenerate eigenmodes with the same frequency as the corresponding single QW, which is similar to bounds states in multiple QW [20]. When inter-well spacing is small enough, the degeneracy of eigenmodes tend to disappear which leads to N-fold splitting owing to the inter-well coupling. In Figure 6, we calculate the transmittance through the $[(AB)_m(CD)_2]^2(AB)_m$ systems with different period numbers for PCs AB. One can discover from Figures 6a–6c that the interval of frequency splitting becomes larger with the period number of $(AB)_m$ reducing. This is due to the fact that decreasing the period number of $(AB)_m$ means decreasing the inter-well spacing. This causes the coupling strength of inter-well to increase which results in a much larger splitting interval.

In order to better understand the inter-well coupling mechanism which results in the modes splitting, it is necessary to calculate the electric field intensity in

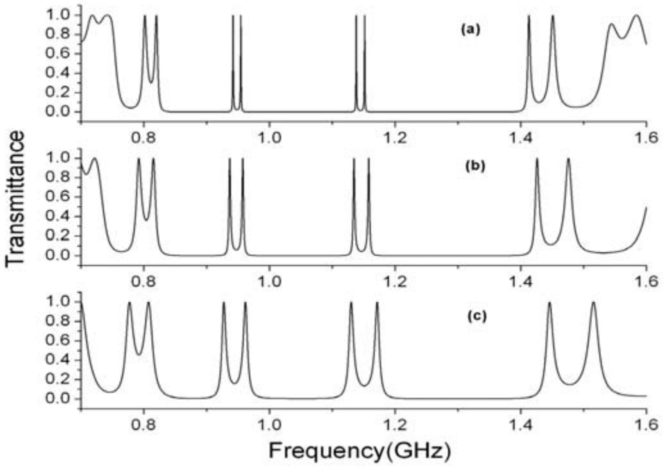

Fig. 6. The transmittance through the $[(AB)_m(CD)_3]^2(AB)_m$ structure: (a) for $m = 4$, (b) for $m = 3$ and (c) for $m = 2$, respectively. Other parameters are the same as those in Figure 2.

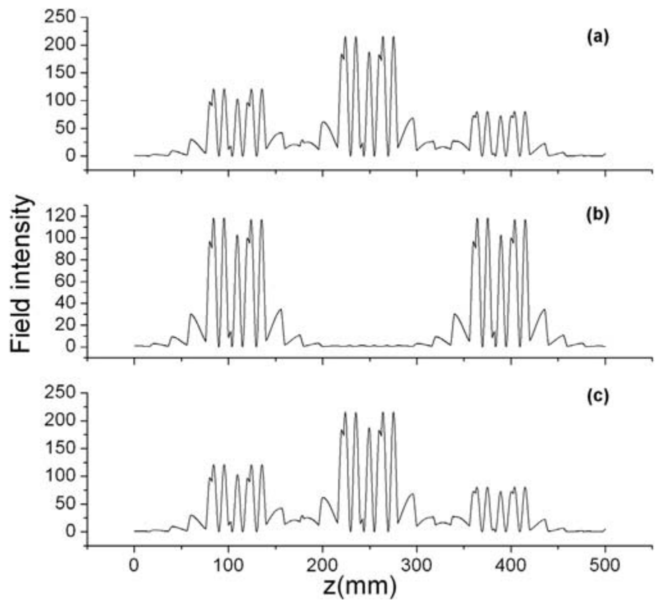

Fig. 7. The electric field intensity inside the $[(AB)_4(CD)_3]^3(AB)_4$ structure at normal incidence. Corresponding resonant frequencies are: (a) 1.034 GHz, (b) 1.041 GHz and (c) 1.045 GHz. Other parameters are the same as those in Figure 2.

multiple photonic QW. With the help of transfer matrix methods, we show the electric field intensity $|E|^2$ of the $[(AB)_4(CD)_3]^3(AB)_4$ structure in Figures 7a–7c. Corresponding frequencies are 1.034 GHz, 1.041 GHz and 1.045 GHz which are the second three-fold splitting modes located in the middle of Figure 5c. It can be seen from Figure 7a and 7c that the field intensity of the first and the third splitting modes localised strong in the well regions, especially in the second well. For the second spitting mode, the electric field mainly localised the first and the third well as shown in Figure 7b.

4 Conclusions

In conclusion, resonant modes in the zero-$\bar{n}$ gap of photonic QW with NIMs were investigated. It is shown that the number of resonant modes is equal to that of the period in the well region and that the resonant modes are insensitive to incident angle and polarisation. We then calculated the transmittance of multiple photonic QW. The result indicated that the resonant modes undergo N-fold splitting due to inter-well coupling. Furthermore, the splitting interval can be adjusted by changing the thickness of photonic barriers.

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