On the time evolution of the cloaking effect of a metamaterial slab

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We investigated the time evolution of the cloaking behavior of a small particle placed in front of a meta-material slab with $\varepsilon = \mu = -1 + i\delta$. We found that the dipole excitation would be suppressed in the long time limit. While on the way to being cloaked, the excitation will exhibit oscillatory behavior as the result of the interference between particle-slab resonances and high density-of-states surface modes. © 2012 Optical Society of America

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The possibility of making objects invisible has captured the imagination of many people for a long time, but it was only very recently that the method of transformation optics provided a systematic way to design invisibility cloaks [1–3]. The cloaks designed by transformation optics typically require complex metamaterial structures (see, e.g., [4]). It is less well known that a small object is automatically cloaked when it is placed close to a "super lens," defined as a slab with $\varepsilon = \mu = -1 + i\delta$ and δ is a small number [5,6]. First discovered by Milton and co-workers in the electrostatic limit [7,8], it can be shown [7,9] that the dipole excitation of a small particle cannot be excited by an external source if the particle is placed within a distance of $z_d < d/2$ where d is the thickness of the super lens. As the dipole response cannot be excited, the small particle is not visible (i.e., cloaked). Chance *et al.* have also studied the similar problem of the lifetime of molecular fluorescence near a metal/dielectric interface [10]. However, most of the previous studies on the cloaking effect or the imaging properties of metamaterial slabs correspond to a steady state solution. As all metamaterials are dispersive, we expect complex time evolution behavior if the source has a switch-on process. This is the purpose of this article to investigate the time evolution of the cloaking behavior.

Let us consider a system consisting of a small particle placed in front of a metamaterial slab (with thickness $d = 0.1$ m) at $(0, 0, z_d)$ and illuminated by a y-polarized (parallel to surface) point current source located at $(0, 0, z_{\mathrm{src}})$ and described by $\mathbf{J}_{\mathrm{source}}(r, t) = \hat{y} J_0 \delta(\mathbf{r} - z_{\mathrm{src}}\hat{z}) \exp(-i\omega_c t)\theta(t)$, which has a switch-on time process $\theta(t) = 1/[1 + \exp(-At)]$, $(A > 0)$. We fix $z_{\mathrm{src}} = 0.15$ m which is larger than the half width of the slab. The metamaterial slab region is from $z = 0$ to $z = -d$ and infinite along x and y directions. The relative permittivity and permeability of the slab are $\varepsilon_s(\omega) = \mu_s(\omega) = 1 + \omega_{sp}^2/(\omega_{sr}^2 - \omega^2 - i\nu\omega)$, where $\omega_{sp} = 6.921 \times (10^{10}\ \mathrm{rad/s})$, $\omega_{sr} = 2.825 \times (10^{10}\ \mathrm{rad/s})$ and $\nu = 2.119 \times (10^4\ \mathrm{rad/s})$. At the working frequency, $\omega_c = 5.561 \times (10^{10}\ \mathrm{rad/s})$, $\varepsilon_s(\omega_c) = \mu_s(\omega_c) = -1 + i\delta$ with $\delta = 10^{-6}$. For convenience of discussion, we define a dimensionless parameter $t_s = (t_2 - t_1)/T_0$ to characterize the speed of switch-on process, where t_2 and t_1 are defined by $\theta(t_2) = 0.99$, $\theta(t_1) = 0.01$ and $T_0 = 2\pi/\omega_c$. The radius, relative permittivity, and permeability of the particle are set at $a = 2$ mm, $\varepsilon_p(\omega) = 1 + \omega_{pp}^2/(\omega_{pr}^2 - \omega^2 - i\gamma\omega)$, and $\mu_p(\omega) = 1$, where $\omega_{pp} = 2.0 \times (10^{10}\ \mathrm{rad/s})$, $\omega_{sr} = 2.825 \times (10^{10}\ \mathrm{rad/s})$ and $\gamma = 5.0 \times (10^8\ \mathrm{rad/s})$. In the frequency region of our interest, the particle satisfies the condition $\sqrt{\varepsilon_p(\omega)\mu_p(\omega)} k_0 a \ll 1$, where $k_0 = \omega/c$. We only consider the electric dipole response of the particle and the polarizability of the particle is $\alpha(\omega) = i3a_1/(2k_0^3)$, where a_1 is the electric dipole term of the Mie coefficients. The exact values of these parameters do not affect the salient features of our discussion below as long as we have a dispersive system satisfying $\varepsilon_s = \mu_s = -1 + i\delta$ at a particularly working frequency ω_c and δ is small so that the slab can be considered as a "super-lens" and the particle is small in size ($a \ll \lambda$). To visualize the time evolution of the cloaking process, we will study the induced dipole moment $\boldsymbol{P}(t)$ of the particle as the source is turned on. We will normalize $\boldsymbol{P}(t)$ by $|\boldsymbol{P}_0|$, which is the induced dipole moment of the particle in the absence of the super lens in the $t \to \infty$ limit. If $\lim_{t\to\infty} |\boldsymbol{P}(t)| \ll |\boldsymbol{P}_0|$, we can claim that the particle has been cloaked; otherwise an image will emerge on the other side of the super lens.

Because the external source is polarized along the y direction, the nonzero component of $\boldsymbol{P}(t)$ is also oriented along the y direction by symmetry. The induced dipole moment in the frequency domain can be written as

$$P_y(z_d; \omega) = \frac{\alpha(\omega)}{1 - 4\pi\alpha(\omega) k_0^2 W_{yy}^{\mathrm{ref}}(z_d; \omega)} E_y^{\mathrm{ext}}(z_d; \omega), \qquad (1)$$

where E_y^{ext} is the electric field at the particle due to the external source, including the field directly from the source and that reflected from the slab. The function $W_{yy}^{\mathrm{ref}}(z_d; \omega)$ is one component of the reflection part of the dyadic Green's function, which is given in [9].

Using a time-dependent Green's function method [11], the y component of the induced dipole moment $P_y(z_d; t)$ can be written as:

$$P_y(z_d; t) = \frac{1}{2\pi} \int \mathrm{d}\omega e^{-i\omega t} P_y(z_d; \omega)$$
$$= \lim_{\eta\to 0} \frac{1}{2\pi} \int \mathrm{d}\omega e^{-i\omega t} \tilde{P}_y(z_d; \omega) H(\omega), \qquad (2)$$

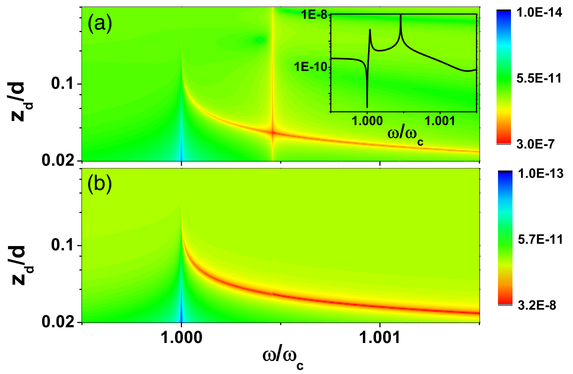

Fig. 1. (Color online) (a) Induced dipole moment of the passive particle $|\tilde{P}_y(z_d;\omega)|$ and (b) response kernel $|\alpha(\omega)/[1-4\pi\alpha(\omega)k_0^2 W_{yy}^{\text{ref}}(z_d;\omega)]|$ as functions of z_d/d and frequency ω/ω_c. The inset in (a) shows $|\tilde{P}_y(\omega)|$ as a function of frequency ω/ω_c at the value $z_d/d = 0.08$. The absorption of the slab is $\delta \sim 10^{-6}$.

where the frequency dependence of the kernel $H(\omega) = 2\pi\omega \exp[-\pi(\omega-\omega_c+i\eta)/A]/\{A[1-\exp[-2\pi(\omega-\omega_c+i\eta)/A]]\}$ originates from the time dependence of the source and $\eta > 0$ is an infinitesimal number to ensure causality. In the steady state limit $(t\to\infty)$, $\lim_{t\to\infty} P_y(z_d;t) = -i\tilde{P}_y(z_d;\omega_c)\omega_c \exp(-i\omega_c t)$. The dissipation will eventually damp out all the transients due to the switch-on process, leaving only the working frequency mode to drive the system.

We plot the excited dipole amplitude in frequency domain $\tilde{P}_y(z_d;\omega)$ as a function of z_d and ω in Fig. 1(a) and the response kernel $|\alpha(\omega)/[1-4\pi\alpha(\omega)k_0^2 W_{yy}^{\text{ref}}(z_d;\omega)]|$ in Fig. 1(b). A blue color indicates a small number while red indicates a big number. We see from Fig. 1(a) that $\tilde{P}_y(z_d;\omega)$ is very small at $\omega = \omega_c$ for all $z_d < d/2$, and the value gets smaller for smaller values of z_d. Comparing with Fig. 1(b), we note that this suppression comes from the response kernel, and this happens because the $|W_{yy}^{\text{ref}}(z_d;\omega)|$ becomes very large at ω_c. It can be shown that $|W_{yy}^{\text{ref}}(z_d;\omega)| \sim 1/[(1-\omega/\omega_c)(z_d/d)^3]$ in the limit $z_d/d \to 0$, $\omega\to\omega_c$ if δ is small. This suppression effect actually accounts for the "cloaking effect" of the super lens in the steady state limit [7]: In the $t\to\infty$ limit, all the transients would have been dissipated, and the very small values of $\tilde{P}_y(\omega_c)$ mean that the dipole cannot be excited by the external field and hence it cannot be detected (i.e., "cloaked").

However, there are two prominent "red" regions in Fig. 1(a) that signify strong excitation, and these excitations govern the time evolution behavior of the cloaking process. One of these resonances, which satisfies the condition $|1-4\pi\alpha(\omega)k_0^2 W_{yy}^{\text{ref}}(z_d;\omega)| \approx 0$, also shows up clearly in Fig. 1(b). This is a resonance due to the coupling of the particle with the slab, and the resonance frequency varies as $(\omega/\omega_c - 1) \propto (z_d/d)^{-3}$ when z_d/d and $(\omega/\omega_c - 1)$ are small numbers. The resonance moves away from ω_c as $z_d/d \to 0$, but we see from Figs. 1(a) and 1(b) there is a range of z_d within which the resonance frequency is close to ω_c. The other strong dipole excitation is independent of z_d and has a frequency slightly above ω_c. This feature does not show up in Fig. 1(b), showing that it comes from E_y^{ext}, which is the field strength at the position of the particle. This is due to the surface waves of the metamaterial slab. The coupling of the surface waves on either side of the metamaterial slab splits the surface modes into two branches, one above and one below ω_c. In the upper branch, there is always a point where the group velocity of the surface wave is zero, which gives a huge density of states (DOS). If the external source has a frequency component corresponding to the high DOS modes, these high DOS modes will be strongly excited, giving rise to an enhanced field $|E_y^{\text{ext}}|$ at the position of the particle and driving a big $\tilde{P}_y(z_d;\omega)$ that shows up as a peak at $\omega \to 1.00046\omega_c$ in the inset in Fig. 1(a), where we plot $|\tilde{P}_y(\omega)|$ for $z_d = 0.08d$. From the inset, we clearly see the dip ("cloaking" effect) and two resonance peaks due respectively to particle-slab resonance and high DOS surface modes in the slab. These two resonance peaks will interfere with each other, and as their frequencies are close to each other, their beating will have a long period.

The time evolution of the cloaking effect is shown in Fig. 2, where we plot $|P_y(t)|$ normalized by $|\boldsymbol{P}_0|$ for different values of z_d. In the calculations, we set $\delta = 10^{-6}$, $t_s = 1$. We first focus on the results for $z_d = 0.04d$ (blue curve). When the dipole is placed at this distance, the surface mode and particle-slab resonance mode are both excited [see Fig. 1(a)], and their beating leads to the

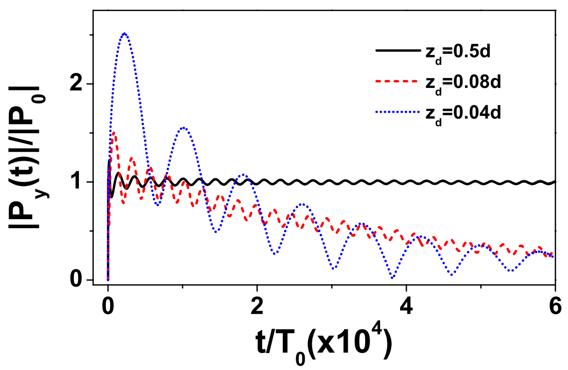

Fig. 2. (Color online) Time evolution of the normalized induced dipole moment of the particle $|P_y(t)|/|\boldsymbol{P}_0|$ for different values of z_d (distance between the particle and the slab) for $\delta = 10^{-6}$, $t_s = 1$.

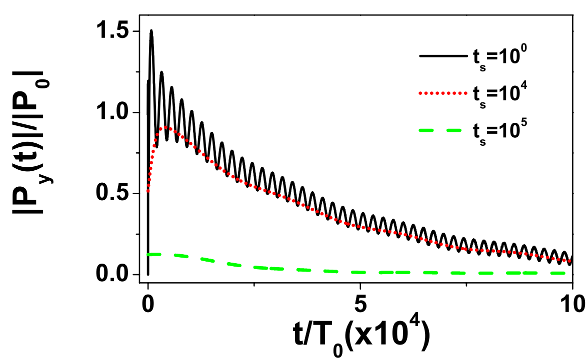

Fig. 3. (Color online) Time evolution of normalized induced dipole moment of the particle $|P_y(t)|/|\boldsymbol{P}_0|$ for different switch-on times of the current source t_s. The absorption of the slab and the distance between the particle and the slab are $\delta = 10^{-6}$, $z_d/d = 0.08$.

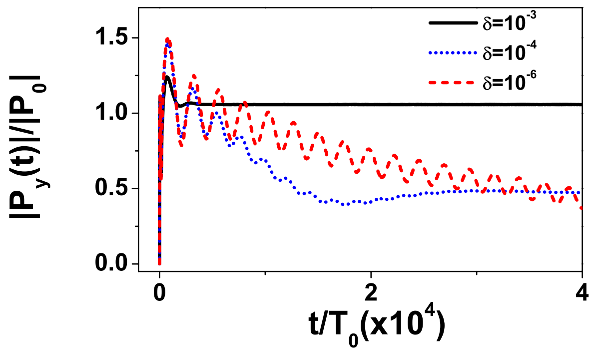

Fig. 4. (Color online) Time evolution of normalized induced dipole moment of the particle $|P_y(t)|/|\boldsymbol{P}_0|$ for different absorption parameters δ. We set $t_s = 1$, $z_d/d = 0.08$.

oscillatory behavior in $|P_y(t)|$. The dissipation damps out all transients due to $P_y(\omega \neq \omega_c)$, leaving the response due to the working frequency $\omega = \omega_c$ in the long time limit. As $\tilde{P}_y(\omega_c)$ is small [see Fig. 1(a)], the particle is "cloaked" eventually, but on its road to being cloaked, the values of $|P_y(t)|$ can actually be a few times that of $|\boldsymbol{P}_0|$. When the particle is placed further from the slab, at $z_d = 0.08d$, the oscillations becomes shorter in period (red curve). From Fig. 1(a), we see that the particle-slab resonance is further away from the high DOS surface mode, leading to a higher beating frequency. It is known that the cloaking effect of the super lens only works for $z_d < d/2$ [7,9]. As a check, we show results for the critical value of $z_d = 0.5d$, and there is indeed no cloaking as $|P_y(t)| \approx |\boldsymbol{P}_0|$. There is a small oscillation due to interference of the working frequency with the high DOS surface modes of the slab.

In Fig. 3, we show the induced dipole moment for different values of t_s. Here, $\delta = 10^{-6}$, $z_d = 0.08d$. For $t_s = 1$, the oscillation is obvious while for $t_s = 10^4$ and 10^5 the oscillations are hardly noticeable. For $t_s = 1$, the turn-on process is more abrupt, and the frequencies produced by the source will cover all the whole frequency range shown in inset figure in Fig. 1(a). Hence the oscillations appear as we have discussed in the previous section. For $t_s = 10^4$ and 10^5, these "gentle" switch-on processes introduce frequency components that are very narrowly squeezed at the working frequency ω_c, and the resonance modes shown in Fig. 1(a) are very weakly excited. The oscillations are weak, and the time evolution is basically manifesting the absorption of the materials.

The effect of absorption of slab is plotted in Fig. 4, where we fix $t_s = 1$, $z_d = 0.08d$ and we have chosen three different values of absorption, namely $\delta = 10^{-3}$, 10^{-4}, 10^{-6} to illustrate the effect of absorption. When the absorption is small ($\delta < 10^{-5}$), the oscillatory behavior is very conspicuous, and $|P_y(t)|$ decays in an oscillatory manner to a small value ("cloaked"). When the absorption reaches a higher value of $\delta \sim 10^{-3}$, there is hardly any oscillation and no "cloaking." When the absorption reaches an intermediate value, of $\delta \sim 10^{-4}$, the cloaking effect becomes observable. Also, with a smaller absorption, the dipole moment reaches steady state more slowly.

In summary, we investigated the time evolution of the cloaking process of a small particle placed close to a metamaterial slab with $\varepsilon_s(\omega_c) = \mu_s(\omega_c) = -1 + i\delta$. We show that there are resonance states in the particle-slab system that are very close in frequency to the working frequency of the "super lens," and it is very likely that these resonance states will be excited during a switch-on process and the beating of these resonances will affect the time evolution of the cloaking process. The dependence of such time evolution behavior on various system parameters is discussed.

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