Image Formation of an Object under External Illumination in Front of a Metamaterial Slab

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***Abstract*- We will examine the issue of image formation of an object placed in front of a negative index slab, and see under what circumstances should we see an image and under what circumstances would the object become invisible and thus no image is formed.**

I. INTRODUCTION

Pendry [1] showed that a slab with $\varepsilon = \mu = -1$ (lossless Veselago slab [2]) can focus all the Fourier components of light including the evanescent components which makes the Veselago slab [2] a perfect lens. The concept of complementary media [3] can be used to understand the image formation. In particular, it states a slab with (ε, μ) and a slab of equal thickness constructed to be an inverted mirror image and (ε, μ) reversed in sign is equivalent to an optical void. This immediately explains why the Veselago lens is a perfect lens. However, Milton *et. al.* [4, 5] showed that a finite loss of the Veselago lens will induce localized resonance which will influence the image formation for objects that lie within the resonance region. As the loss approaches zero, a point object located less than a specific distance from the lens cannot be excited by external field and thus the slab cannot form an image.

In this talk, we will examine the image formation of an object located in front of a metamaterial slab with arbitrary material composition, and to see under what circumstances would the slab behave as a lens or as a cloak.

II. THE FORMALISM

We consider a small object placed at the distance z_d in front of a metamaterials slab with material parameters (ε, μ) and thickness d. The object is described a dynamic dipole polarizability $\alpha = i\,(3/2k_0^3)\,a_1$, where $k_0 = \omega/c$ with c as the speed of light, and a_1 is the electric term of the Mie's coefficients. We assume that the external light source is a dipole source with a polarization along the y direction (parallel to the slab), and both of the active dipole and the passive object are placed at z axis, the nonzero component of the induced dipole moment is oriented in y direction so that

$$p_y = 4\pi k_0^2 \alpha^* W_{yy,dip}^{tot} p_{src}, \tag{1}$$

where p_{src} is the external dipole source. $W_{yy,dip}^{tot}$ is the yy-element of the dyadic Green's function at the passive dipole which takes into account the effect of the slab. α* is the effective polarizability with the form of,

$$\alpha^* = \left(\alpha^{-1} - 4\pi k_0^2 W_{yy}^{ref}\right)^{-1}. \tag{2}$$

Here W_{yy}^{ref} is the yy-element of the reflection part of the dyadic Green's function, with the form of,

$$W_{yy}^{ref} = \frac{i}{8\pi}\int_0^\infty \frac{k_{//}}{k_{0z}} dk_{//}\left(R^{TE} e^{i2k_{0z}z_d} - \frac{k_{0z}^2}{k_0^2} R^{TM} e^{i2k_{0z}z_d}\right), \tag{3}$$

where R^{TE} and R^{TM} are the reflection coefficients of the slab [6]. Also, by using Eq. (1), the electric field at an arbitrary point $\mathbf{R}$ can be written as,

$$E_y(\mathbf{R}) = \omega^2 \mu_0 \left[W_{yy,\mathbf{R}}^{tot} + \alpha^* 4\pi k_0^2 W_{yy,\mathbf{R}}^{tran} W_{yy,dipole}^{tot}\right] p_{src}, \tag{4}$$

where $W_{yy,\mathbf{R}}^{tot}$ represents the yy-element of the total Green's function radiated at $\mathbf{R}$ by the external source, and $W_{yy,\mathbf{R}}^{tran}$ represents the yy-element of the Green's function radiated at $\mathbf{R}$ by the dipole object [6].

III. EXCITATION OF THE DIPOLE FOR ISOTROPIC NEGATIVE INDEX SLAB

Eq. (2) can be interpreted as the single particle dipole polarizability that includes the reflection response of the slab. It is as if the object has the effective polarizability. When α* → α, we can regard as the object being completely excited, and the slab behaviors potentially as a lens because the dipole becomes a source of energy and if the slab has negative index, it can focus the energy into an image behind the lens. However, if α* → 0, the slab serves as a cloak because the induced dipole moment vanishes, and the total field behind the slab is the same as if the object is not there. The functionality of the slab as a lens or as a cloak can be seen by examining α*.

We shall now examine the asymptotic behaviors of W_{yy}^{ref}. When ε and μ are not -1, we can show that

$$W_{yy}^{ref} \to \frac{1}{16\pi}\left[R_{\lim}^{TM}\left(\frac{\kappa_0^2}{k_0^2 z_d} + \frac{\kappa_0}{k_0^2 z_d^2} + \frac{1}{2k_0^2 z_d^3}\right) + \frac{R_{\lim}^{TE}}{z_d}\right] e^{-2\kappa z_d}, \tag{5}$$

where κ_0, $R_{\lim}^{TE}$, and $R_{\lim}^{TM}$ are constants related to the material parameters of the slab, and the wavevector k_0. Therefore, as $z_d \to 0$, W_{yy}^{ref} diverges as $\exp(-2\kappa_0 z_d) / z_d^3$. It indicates that α* goes to zero smoothly and the excitation will be suppressed as the dipole move closer and closer to the slab. We emphasize that the suppression effect occurs for any material parameter value (except for when $\varepsilon = \mu = 1$) as long as the object is sufficiently close to the slab. Because the big $k_{//}$ component responses of the slab can suppress the dipole excitation (α* → 0) completely as long as the object moves sufficiently close to the slab.

The case of *ε, μ → -1* is special. Fig. 1 shows the numerical simulation results of the cloaking and imaging properties of the lossy Veselago slab when the dipole object is placed at different distances from the slab.

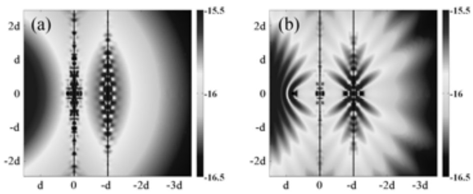

Fig. 1. (a) Cloaking and (b) imaging effect of the lossy Veselago slab ($\varepsilon = \mu = -1 + 10^{-7} i$) when the object locates at z_d = d/5 and 4d/5.

From Fig. 1, one can see that the Veselago slab can behave as a cloak (left) or as a lens (right), depending on the location of the small object. By carefully examining the asymptotic behavior of W_{yy}^{ref} in the limit of $\varepsilon = \mu = -1+i\delta$, $\delta \to 0$, it can be shown that the reflection due to evanescent waves can be approximated as $W_{yy}^{ref} \to C/\delta^{\gamma}$, where C is a constant, and $\gamma = 1 - 2z_d/d$. It is clearly seen that $\lim_{\delta \to 0} W_{yy}^{ref} \to \infty$ if $z_d < d/2$ and $\lim_{\delta \to 0} W_{yy}^{ref} \to 0$ if $z_d > d/2$. It indicates that the Veselago slab with $\varepsilon = \mu = -1+i\delta$ have cloaking effect with a critical distance *d/2* in the zero absorption limit. This is the finite frequency analog of the anomalous resonace effect first noted by Milton *et al.* for two-dimensional line dipole configurations in the quasistatic limit [5].

When $\varepsilon = -1 + i\delta$, $\mu \neq -1$, we find that the cloaking distance has a transition from the quasi-static limit to finite frequency region. In the quasi-static limit, there is a finite "suppression zone" of *d/2* in the limit $\delta \to 0$. While in the high frequency regime, α* smoothly approaches zero with an asymptotic form of z_d^5, and there is no critical distance. In addition, for $\mu = -1 + i\delta$, $\varepsilon \neq -1$, α* smoothly approaches zero with an asymptotic form of z_d^3 so that the suppression is not as strong as in the case with $\varepsilon = -1 + i\delta, \mu \neq -1$.

We also find that suppression zones can also occur in other negative index slabs if we allow for anisotropy. In general, a "folded geometry" slab [7,8] (within the framework of transformation optics) with $\varepsilon = \mu = diag(-\beta, -\beta, -1/\beta)$, $\beta > 0$, the reflection coefficient R for evanescent components is the same as that of a Veselago lens of *βd*. Hence, the cloaking effect also occurs in this kind of anisotropic negative refractive index slab with a critical distance of *βd/2*. Fig. 2 shows the numerical demonstrations of the cloaking effect calculated by the COMSOL package. We see that the object can be cloaked [Fig. 2(a)] or imaged [Fig. 2(b)] by the slab depending on whether is it within or outside $z_d = \beta d/2$.

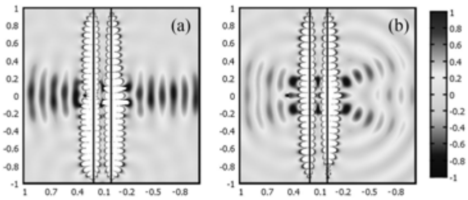

Fig. 2. The "dipole" object (black dot) is (a) cloaked or (b) imaged by an anisotropic "folded geometry" slab.

IV. SUMMARY

In summary, we have investigated the image formation of a dipole object in front of a metamaterial slab. The excitation of a dipole object can be suppressed if it is placed sufficiently close to the slab with arbitrary values of (*ε, μ*). The suppression is strongest in "folded geometry" slabs which have a finite suppression zone of *βd/2* in the limit of small absorption.

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